मण्डलबेथ [maṇḍalabeth] 3D fractals

elizabeth r chen

¡we can construct मण्डलबेथ [maṇḍalabeth] fractals of any dimension!
& any symmetry group G of a bouquet of unit circles.
this is the algorithm for 2D fractals living in 3D euclidean space.
higher mathematically, ¡this is a very beautiful & elegant construction!
the forest is the entire generating function, sum over the symmetry group.
the trees are each term $T^{-1} f\, T\langle x,y,z\rangle$ of the matrix & vector equation.
the leaves are the individual scalar components, which are very messy.

0 (standard) unit circle

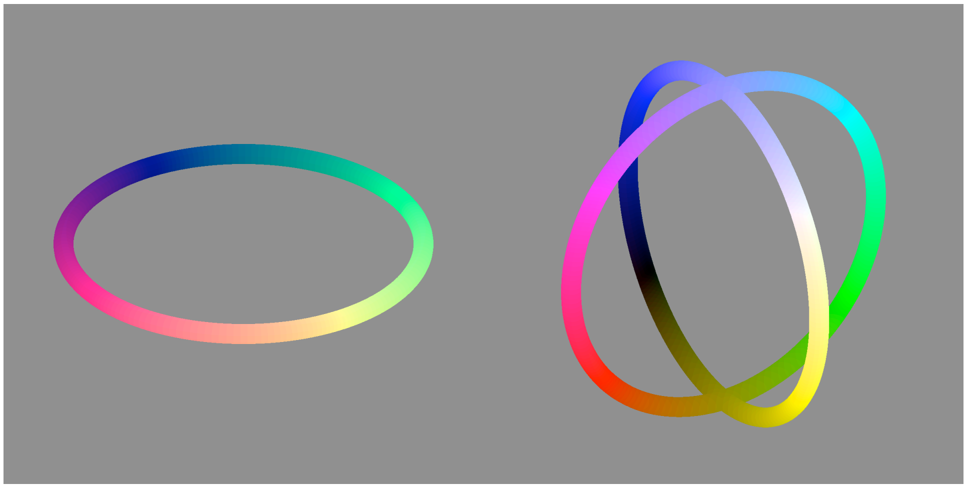
we consider the (standard) unit circle $[x^2 + y^2 = 1] \wedge [z = 0]$
with n equally spaced control points $\langle \cos {}^{2\pi}/_{n}, \sin {}^{2\pi}/_{n}, 0\rangle$.
the (standard) mandelbrot set has generating function
$f\langle x,y,z\rangle = \langle \text{real } (x+iy)^{n+1}, \text{imag } (x+iy)^{n+1}, 0\rangle$.
the symmetry group is the dihedral group D^{2n}.
any symmetry S permutes the control points,
but fixes the circle & the generating function $f\langle x,y,z\rangle$.

1 (rotated) unit circle

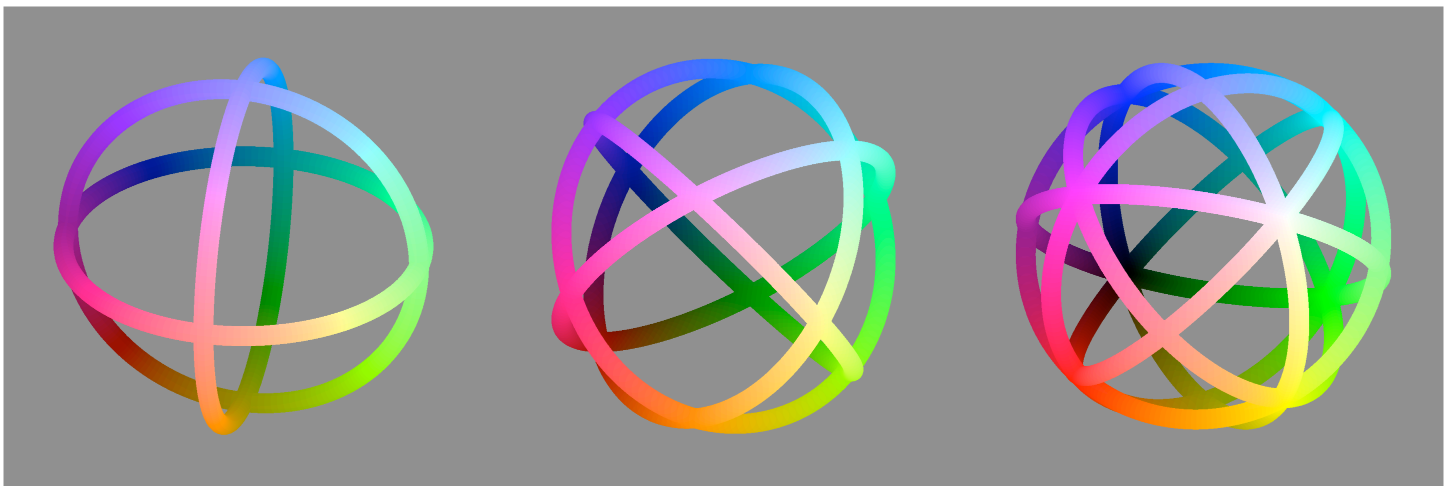
now we generalize to any (rotated) unit circle, which is just a
transformation T of the (standard) unit circle, rotated about the origin.
the (rotated) mandelbrot set has generating function $T^{-1} f\, T\langle x,y,z\rangle$.
the symmetry group is also the dihedral group $T^{-1}D^{2n}T \approx D^{2n}$.
any symmetry $T^{-1}ST$ permutes the control points,
but fixes the circle & the generating function $T^{-1} f\, T\langle x,y,z\rangle$.

2 bouquet of unit circles

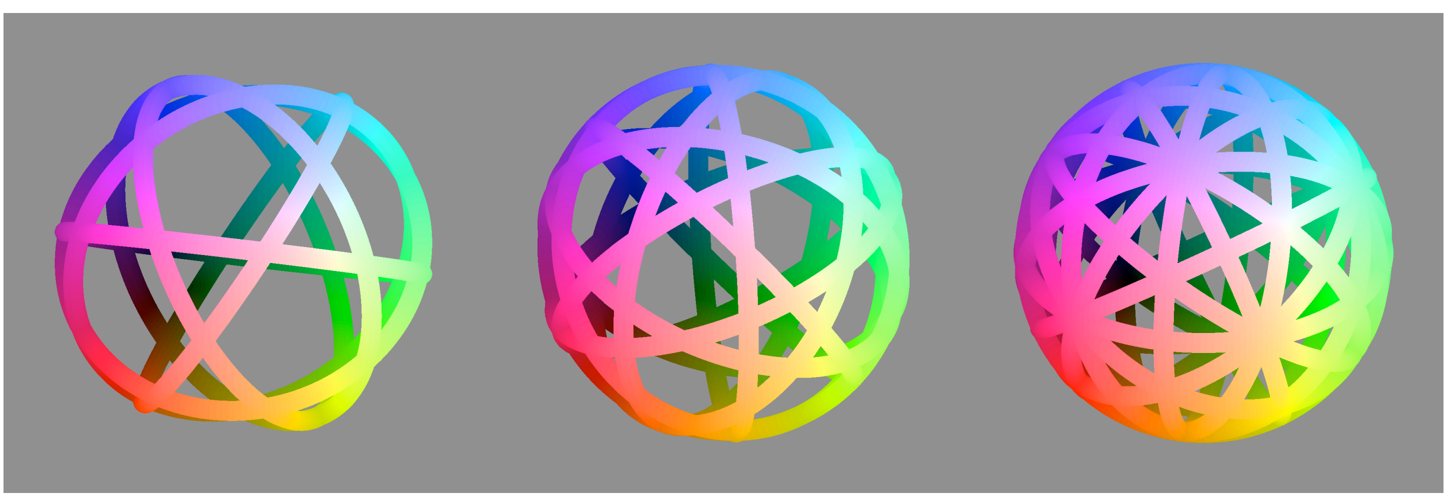
now we consider a bouquet of unit circles & control points,
with symmetry group G. any symmetry can permute
the circles & control points, but fixes the bouquet.
each circle contributes a term of the form $T^{-1} f\, T\langle x,y,z\rangle$.
we add up all the terms from all the circles.
this is the generating function for the मण्डलबेथ [maṇḍalabeth].
any symmetry can permute the terms, but fixes the sum.
so the मण्डलबेथ [maṇḍalabeth] also has symmetry group $T^{-1}GT \approx G$.



M_1^{1n} M_2^{1n}



M_3^{2n} M_4^{3n} M_6^{2n}



M_6^{5n} M_{10}^{3n} M_{15}^{2n}