Comparative Vagueness*

Alex Silk
a.silk@bham.ac.uk

1 Comparative sorites

Positive-form predications ('x is tall', 'x is bald') reign supreme in discussions of linguistic vagueness. It is often assumed in linguistic circles that explicit comparatives — comparatives 'x is ADJ-er (/more ADJ) than y' using a comparative morpheme — are not vague (see also Cooper 1995: 246; Kennedy 2007b: 6, 2011: 74, 82–83, 93, 2013: 271; McNally 2011: 164n.10; van Rooij 2011a: 65–69):

> [T]he comparative form ... is not vague... [A] core semantic difference between the positive [(i.e. unmodified)] and comparative forms ... is that the latter lacks whatever semantic (or pragmatic) features give rise to the vagueness of the former, and simply expresses an asymmetric ordering relation. (Kennedy 2013: 269–270)

> [A]djectives in the comparative are uniformly non-vague. (Bochnak 2013: 42)

Such remarks are generally made in passing in view of "prototypical relative adjectives" (McNally 2011: 163) such as 'tall'. Here again is Kennedy:

> The comparative predicate *taller than David* ... denotes a property that is true of an object just in case its height exceeds David's height. This is a precise property..., since whether it holds of an object or not is fully determined by facts about that object's height. (Kennedy 2013: 270)

Vagueness derives from fuzziness in standards of application — how many millimeters of height one must have to count as tall, how many cents one must have to count as rich, and so on. Hence, "Unsurprisingly, comparatives ... do not give rise to the Sorites paradox, and do not have borderline cases" (McNally 2011: 164n.10).

But they do.

Suppose you must decide between saving your dear friend and saving some number of strangers. Plausibly we have some special obligations to those close to

*Certain ideas in the paper draw on material from Silk 2016: chs. 6–7 and 2021.

us, so that it is morally better — or at least not morally worse — for you to save your dear friend than to save two strangers. But there doesn't seem to be a precise number of strangers that would tip the balance. Consider (1):

(1) (P1) Your saving 2 strangers is not morally better than your saving your dear friend.
(P2) For all $n > 2$, if your saving n strangers is not morally better than your saving your dear friend, then your saving $n + 1$ strangers is not morally better than your saving your dear friend.
(C) $\therefore$ For all $n > 2$, your saving n strangers is not morally better than your saving your dear friend.

No one's friends are that important.

Or suppose you like sugar in your coffee. Yet it's not as if you care exactly how sweet it is. As far as your preferences go, one day's sweetness is as good as any other (at least up to a point, say K; there is, perhaps, such a thing as too sweet). Consider (2) — where x_s is an ordinary sweetened cup of coffee, and $x_1 \ldots x_i \ldots$ is a series of otherwise identical cups differing only in quantity of sugar, with x_i a (pre-K) cup with i micrograms of sugar (cf. Luce 1956).

(2) (P1) x_1 is not tastier than x_s.
(P2) For all n, if x_n is not tastier than x_s, then x_{n+1} is not tastier than x_s.
(C) $\therefore$ For all n, x_n is not tastier than x_s.

But not just any cuppa can be the best.

That is: The premises seem true — in (1), given the nature of morality; in (2), given one's preferences.[1] The arguments seem valid. Yet the conclusions are false. There may be something wrong with sugar in one's coffee, but not that thinking otherwise leads to paradox.

Sorites-susceptibility is one hallmark of vagueness. The comparatives in (1)–(2) exhibit "tolerance": given your preferences, one microgram of sugar is "insufficient to affect the justice with which [the comparative predicate 'is (not) tastier than x_s'] applies" (Wright 1975: 349). Comparatives may also have "fuzzy" or "blurred boundaries of application": "There is, for example, no sharp division be-

[1]What if the moral facts or your preferences are such that one stranger or one microgram of sugar really does tip the balance? Then consider Pat, who thinks otherwise and who doesn't care about differences between adjacent cups, and read 'morally better' as 'morally better according to Pat' and 'tastier' as 'tastier to Pat'. Any compositional semantics will need to be able to characterize such evaluative views and states of mind so as to make sense of such explicit relativizations.

tween [cups] that are clearly [tastier than x_s] and [cups] that aren't" (Raffman 1994: 41). The comparative may have borderline cases — for instance, cups in the "penumbra" (Russell 1923: 87) that are neither clearly tastier than x_s nor clearly not.

Examples such as (1)–(2) can be multiplied. Suppose you are comparing edited versions of a dimly lit photo. x_{250}, the version with a 15% brightness increase, is prettier than the original, x_{100}. But it's not as if every 0.1% change in brightness affects the prettiness of the photo. If you judge x_i to be prettier than x_{100}, your judgment about x_{i-1} should be no different. Yet x_{100} is not prettier than itself.

(3) (P1) x_{250} is prettier than x_{100}.
(P2) For all n, if x_n is prettier than x_{100}, then x_{n-1} is prettier than x_{100}.
(C) $\therefore$ For all n, x_n is prettier than x_{100}.

More generally: Take a broadly evaluative gradable expression 'G' that is tolerant in the context with respect to some property F (e.g., 'tasty' and sweetness, 'pretty' and brightness). Consider two items x_i, x_j in the domain of 'G' that are significantly different in F-ness, such that 'x_i is (not) G-er than x_j' is clearly true. Use x_j as a pivot, applying 'is (not) G-er than x_j' of items incrementally different in F-ness from x_i. Find yourself at an item x_k such that 'x_k is (not) G-er than x_j' is clearly false.

The idea that comparatives can be vague isn't unheard of. Some have suggested that borderline cases of comparatives can arise due to multidimensionality or uncertainty (indeterminacy, indecision, imprecision) about measurement procedures (Williamson 1994: 156, Endicott 2000: 43–45, Keefe 2000: 13–14; cf. Sassoon 2011: 102, 106–111, 2013: 172–178, 209). For instance, if individuals differ in "incommensurate dimensions" (Endicott 2000: 43) of niceness, there may be "no fact of the matter about who is nicer" (Keefe 2000: 13). Yet it has been assumed that "if we consider only one dimension of comparison, and suppose perfect accuracy in measurement, vagueness does affect the extension of the positive form of the adjective, but not that of the comparative" (Égré & Klinedinst 2011a: 10). Not so.

Unlike in previous cases of vague comparatives, the vagueness phenomena in (1)–(3) cannot be reduced to matters of what dimensions are relevant or measurement procedures. Many a monistic indirect consequentialist have countenanced special obligations. The force of (2)–(3) turns precisely on fixing on a particular dimension of taste (sweetness) and beauty (brightness). Moreover, the concern with denying the inductive premises isn't simply that doing so would be unwarranted due to limited powers of discrimination. As Wright (1987: 239–243) shows, indiscriminability between adjacent items in a sorites series is insufficient to generate the paradox.[2] Discriminable though adjacent cups in (2) might be, in quantity of sugar or quality of sweetness, one cup is as good as the next given one's preferences.

Upshot: Linguistic vagueness can be associated not only with how ADJ something needs to be to count as ADJ, but with how ADJ things are.

2 Semantics for gradation

Comparative sorites arguments such as (1)–(3) raise a challenge for traditional formal semantics for gradation. Let's start by examining how the arguments would be formalized in prominent degree-based and non-degree-based frameworks. §3 develops an improved semantics that captures the felt fuzziness of comparatives such as those from §1 and what distinguishes them from comparatives with "prototypical" gradable adjectives such as 'tall'.

2.1 Degree-based semantics

A prominent approach is to treat gradable adjectives — adjectives that can take degree morphemes (e.g., '-er'/'more') and modifiers (e.g., 'very') — as associating items with degrees on a scale (Bartsch & Vennemann 1973, von Stechow 1984, Kennedy 1999, 2007b, Heim 2001, Morzycki 2015). For instance, on a Kennedy-style measure-function implementation, 'tall' denotes a function *tall* from individuals to degrees of height, i.e. the individual's maximal height; 'hot' denotes a function *hot* from individuals to degrees of temperature, i.e. the individual's maximal temperature. Though many theories assume that degrees are isomorphic to rational numbers, a minimal constraint is that the relation $\geq$ on the set of degrees D have the structure of a partial order, i.e. that $\geq$ be a reflexive, transitive, antisymmetric relation on D (Kennedy 1999, 2007b, Barker 2002, Lassiter 2015). Compositional details aside, the comparative (4) says that the maximal degree to which Alice is tall is greater than the maximal degree to which Bert is tall, per (5).

(4) Alice is taller than Bert.

(5) (4) is true iff $\mathit{tall}(\mathit{Alice}) > \mathit{tall}(\mathit{Bert})$

Comparative inductive premises such as (P2) in (2) are entailed by (P2′) and (P3) in (6). (P3) is an instance of what is sometimes called *PI-transitivity*, which is a weakening of transitivity — i.e., if a relation $\gtrsim$ satisfies transitivity, PI-transitivity in (7) is also satisfied, where $>$ and $\sim$ are the strict (asymmetric) and non-strict (sym-

[2] Certain of the comparative sorites examples which I used in earlier work were problematic in failing to appreciate this point (Silk 2016: 198–199, 206). Thanks to Gunnar Björnsson for discussion.

metric) subsets, respectively, of $\gtrsim$. The above semantics renders the interpretation of (P3) as in (8),[3] where $T = [\![\text{tasty}]\!]$ is a function from items to their degree of tastiness, a representation of how tasty they are.[4] So (P3) follows from the transitivity of the relation $\geq$ on the domain of degrees D.

(6) (P1) x_s is tastier than x_1.
(P2′) For all n, x_n is as tasty as x_{n+1}.
(P3) For all a, b, c, if a is tastier than b, and b is as tasty as c, then a is tastier than c. (*PI-transitivity*)
(C) $\therefore$ For all n, x_s is tastier than x_n.

(7) *PI-transitivity*
$$(X > Y \wedge Y \sim Z) \rightarrow X > Z$$

(8) $$\forall x \forall y \forall z \colon \big(T(x) > T(y) \wedge T(y) = T(z)\big) \rightarrow T(x) > T(z)$$

Upshot: (P1) is true. (P2′) in (6) describes your non-obsessiveness about coffee sweetness; one cup is as good as the next given your preferences. (P3) is entailed by the general structure of scales and holds with any adjective denotation. (P2) in (2) encodes these dual properties. Yet the conclusion (C) is false. Hence the paradox.

2.2 Delineation semantics

Let's turn to the other main approach to gradation in formal semantics: delineation semantics ("partial predicate," "inherent vagueness" semantics). Delineation theories treat gradable adjectives, like non-gradable adjectives ('digital'), as ordinary predicates. What distinguishes gradable adjectives is that they are sensitive to a contextual comparison class (Klein 1980, von Stechow 1984, Burnett 2012). In one context using (9) might express that Alice is tall for a basketball player; in another context it might express that she is tall for an American woman. Gradable adjectives denote partial functions partitioning a comparison class CC into a positive extension, a negative extension, and an extension gap (the "borderline cases"), per (10).[5]

---

[3] I assume an "equally good" reading of the equative (cf. Bhatt & Pancheva 2007, Rett 2008).

[4] I use 'measure function' not only for adjectives associated with measurement procedures or numerical units of measurement (e.g., height in inches) but for any mapping which would determine an order on objects. What is important about degrees is that they represent how tasty, tall, etc. things are, and that they can be associated with qualitative orderings on items in adjectives' domains. Nothing of metaphysical significance is implied by things having "degrees" of tastiness, value, etc.

[5] Some theories also invoke a parameter δ for relevant standards (Lewis 1970, Barker 2002), e.g. where the positive extension of 'tall' is the set of individuals in CC whose height is at least the standard of tallness δ_{tall}.

(9) Alice is tall.

(10) $[\![\text{tall}]\!]^{CC} = \lambda x : \neg gap_{CC}(tall)(x) \,.\, x$ is tall in CC

The positive-form predication in (9) is true given a comparison class CC iff Alice counts as tall in CC. Following Klein 1980, a comparative such as (4) is true iff there is some CC' such that Alice is tall in CC' and Bert is not tall in CC'.

To avoid problematic entailments, delineation theories impose qualitative restrictions on comparisons among individuals across comparison classes (Fine 1975, Klein 1980, Fara 2000). For instance, if x counts as tall in some CC and x has a greater height than y, then there is no CC' in which y counts as tall and x doesn't. Delineation theorists prove that the qualitative restrictions derive a preorder (reflexive, transitive relation) $\succcurlyeq_A$ "at least as ADJ as" on the set of individuals in the domain of 'ADJ', for any adjective 'ADJ' (van Benthem 1982, Klein 1991, van Rooij 2011a). Degrees and scales may be derived from these qualitative orderings (Cresswell 1977, Bale 2008): The set of degrees D is the set of equivalence classes under $\succcurlyeq_A$; and the relation $\geq_A$ on D is defined accordingly such that $[x]_A \geq_A [y]_A := x \succcurlyeq_A y$ (where $[a]_A$ is an equivalence class $\{b : b \succcurlyeq_A a \wedge a \succcurlyeq_A b\}$). Upshot: The interpretation of any adjective relies on a preorder $\succcurlyeq_A$ on the set of individuals. The transitivity of $\succcurlyeq_A$ again validates (P3), and the paradox is off and running.

2.3 Taking stock

It is common to locate the problem in sorites arguments with positive-form predicates such as (11)–(12) in the inductive premise. (Let x_n be someone who is $4' + n$ nanometers tall.)

(11) (P1) Someone who is $4'$ isn't tall (for a pro basketball player).
(P2) If someone who isn't tall (for a pro basketball player) grows one nanometer, they still won't be tall (for a pro basketball player).
(C) $\therefore$ No one is tall (for a pro basketball player).

(12) (P1) x_0 is not tall.
(P2) For all n, if x_n is not tall, then x_{n+1} is not tall.
(C) $\therefore$ For all n, x_n is not tall.

For instance, even if we can't point to any instance of (P2) in (12) that isn't true, perhaps we can know that it isn't true in any context (Soames 1999, Fara 2000), or no matter what formally precise language we might be speaking (Lewis 1970), or no matter how the conversation might evolve (Shapiro 2006), or on any competent way of applying 'tall' (Kamp 1981, Raffman 2014). Can one not say the same about the generalizations in (1)–(3)?

There is a key difference between the positive-form and comparative-form inductive premises. Consider the degree semantics from §2.1. The positive form is treated as relating a degree to a relevant threshold, or *degree standard*. To a first approximation, (9) is true iff the degree to which Alice is tall, $tall(Alice)$, is at least the degree standard of tallness, s_{tall}, i.e. how tall one must be to count as tall.[6] The challenge for theories of vagueness rejecting (P2) from (12) is to explain why speakers find it compelling even if the predicted truth condition (=(13)) is false at any point of evaluation. For instance, perhaps the falsifying instance is never where one is looking (Fara 2000), or there is uncertainty about what standard is determined by the conversational situation (Barker 2002), or the speakers are undecided about what standard to accept for purposes of conversation (Silk 2016).

(13) $\forall n\colon tall(x_n) \ngeq s_{tall} \rightarrow tall(x_{n+1}) \ngeq s_{tall}$

However, the traditional frameworks for gradation in §§2.1–2.2 treat the generalization (P3) from (6) as trivially *true*—in virtue of the basic structure of scales $\langle D, \geq \rangle$, per degree-based semantics, or the qualitative ordering $\succcurlyeq_A$ on the set of individuals, per delineation semantics. That leaves (P2′). Yet (P2′) also seems true in the context, given one's preferences. Indeed even a supertaster could accept it; one simply doesn't care exactly how sweet the coffee is.

(P2′) For all n, x_n is as tasty as x_{n+1}.

It would be surprising if one could rebut fans of special obligations or sugar-taking coffee drinkers with facts about semantic scale structure.

We need a semantics for gradation that avoids validating generalizations such as (P3) in (6) as a matter of conventional meaning. The semantics developed in what follows provides a basis for denying the inductive premises (P2) in (1)–(3) while maintaining tolerance claims such as (P2′) in (6). I welcome the development of alternatives with which the account may be compared.

3 Semiorders in a degree semantics

We need a semantic structure that permits a certain intransitivity with gradable expressions. A natural way to allow for this is to move from thinking of the relation

[6] Many degree theories derive the positive form by combining the adjective with a null morpheme POS to yield a predicate of individuals (von Stechow 1984, Kennedy 1999).

on the set of degrees or individuals as at least a partial order (§2.1) to thinking of the relation as a *semiorder*, instead. Semiorders have been used fruitfully in measurement theory, choice theory, and mathematical psychology for representing intransitive indifferences.[7] (Indeed, example (2) is adapted from an example from Luce 1956.) The broader research on semiorders provides an independently motivated resource to incorporate into the semantics of gradation.

3.1 Semiorders

Formally, a semiorder $\succsim$ on a set S is an interval order — a reflexive, Ferrers binary relation — that satisfies semitransitivity ((14)); equivalently, $\succsim$ is a semiorder iff there is a real-valued function f and fixed positive number ϵ such that $x \succsim y$ iff $f(x) \geq f(y) - \epsilon$, for all $x, y \in S$ (n. 7).[8]

(14) *Reflexive:* $\forall x \colon x \succsim x$
Ferrers: $\forall x, y, z, w \colon (x \succsim y \wedge z \succsim w) \rightarrow (x \succsim w \vee z \succsim y)$
Semitransitive: $\forall x, y, z, w \colon (x \succsim y \wedge y \succsim z) \rightarrow (x \succsim w \vee w \succsim z)$

Intuitively speaking, semiorders generalize weak orders by comparing items under a "range of fuzziness," representable by uniform-length intervals. The Ferrers property ensures the interval representation, whereby $x \succeq y$ iff x and y can be associated with intervals X, Y, respectively, such that Y doesn't wholly follow X (i.e., $x_i \geq y_i$, for some $x_i \in X, y_i \in Y$). Semitransitivity implies that this can be done with all the intervals having the same length; the "fuzziness," represented by ϵ, is the same for each item (cf. Fishburn 1970: 212, 1973: 93). Strict $\succ$ and non-strict $\sim$ subsets of $\succeq$ can be defined in the usual way: $x \succ y$ iff $x \succeq y \wedge y \not\succeq x$ iff $f(x) - f(y) > \epsilon$; and $x \sim y$ iff $x \succeq y \wedge y \succeq x$ iff $|f(x) - f(y)| \leq \epsilon$ (again for all $x, y \in S$ and some real-valued function f and positive number ϵ).

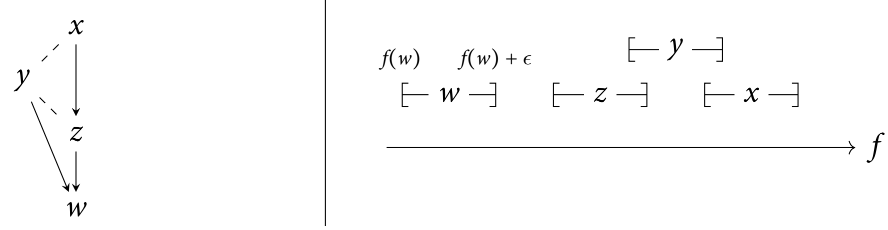
Here is an example. Consider the semiorder $\succeq$ in (15). Alternative graph and interval representations are below, where an arrow $\longrightarrow$ from u to v in the graph represents that $u \succ v$, and a dashed line between them represents that $u \sim v$ (reflexive $\sim$-loops and transitive $\succ$-arcs omitted). An item u is $\succ$-better than v in the graph just in case the fuzziness around u, represented by $[f(u),\ f(u) + \epsilon]$ in the interval representation, strictly follows the fuzziness around v, represented by $[f(v),\ f(v) + \epsilon]$.

[7] For classic discussion see Luce 1956, Scott & Suppes 1958, Fishburn 1985; see also Suppes et al. 1989: ch. 16, Pirlot & Vincke 1997, Aleskerov et al. 2007.

[8] For present purposes we can assume that the sets are finite. For results generalizing to arbitrary sets, see Bouyssou & Pirlot 2021a,b.

(15) $S = \{x, y, z, w\}$
$\geq\ = \{\langle x, x\rangle, \langle x, y\rangle, \langle x, z\rangle, \langle x, w\rangle,$
$\langle y, x\rangle, \langle y, y\rangle, \langle y, z\rangle, \langle y, w\rangle,$
$\langle z, y\rangle, \langle z, z\rangle, \langle z, w\rangle,$
$\langle w, w\rangle\}$

Graph representation: | Interval representation:



Intuitively, the semiorder $\geq$ in (15) weakens an order in which $x > y > z > w$ by comparing items under a "fuzziness" (represented by ϵ) that fails to distinguish x and y and fails to distinguish y and z. Crucially, the non-strict part $\sim$ is no longer an equivalence relation. In (15) we have $\overset{\curvearrowright}{x \sim y \sim z}$; $x \sim y$ and $y \sim z$ but $x \not\sim z$, indeed $x \succ z$. This intransitivity of $\sim$ will be important in what follows.

3.2 Degree semantics

Let's turn to the semantics. To fix ideas I assume a Kennedy-style degree-based framework. The denotations of adjectives 'ADJ' may still be understood as associating items with degrees, conceived as points on a scale; yet a scale is now $\langle D, \succsim_A\rangle$, with $\succsim_A$ a semiorder on the set of degrees D.[9] Truth conditions for the comparative and equative are in (16)–(17) (n. 3), where *adj* is the measure function denoted by 'ADJ', and f_A and ϵ_A are a real-valued function and positive number, respectively, such that $x \succsim_A y$ iff $f_A(x) \geq f_A(y) - \epsilon_A$, for all $x, y \in D$.

(16) '*a* is ADJ-er than *b*' is true
iff $adj(a) \succ_A adj(b)$
iff $f_A(adj(a)) - f_A(adj(b)) > \epsilon_A$

(17) '*a* is as ADJ as *b*' is true
iff $adj(a) \sim_A adj(b)$
iff $|f_A(adj(a)) - f_A(adj(b))| \leq \epsilon_A$

[9]We can assume that the semiorders $\succsim_A$ are also antisymmetric, i.e. $(x \geq_A y \wedge y \geq_A x) \rightarrow x = y$, for all $x, y \in D$. Multiple items may be mapped to the same degree.

One way of interpreting the formalism is to understand the function f_A as mapping degrees of ADJ-ness to measures of a property on which ADJ-ness supervenes — e.g., mapping degrees of tastiness to measures of sweetness (cf. §1). The value ϵ_A can be interpreted as a threshold of distinguishability, or how much more of the subvening property a must have than b in order to count as ADJ-er. Take 'tasty'. If quantity of sugar alone determined tastiness, $f_T(T(a))$ would be a's measure of sugar, and ϵ_T would represent a level of sugar sufficing to distinguish items in how tasty they are. In such a scenario, $T(a) \succ_T T(b)$ iff the difference in sugar between a and b is greater than ϵ_T, and a is tastier than b; and $T(a) \sim_T T(b)$ iff the difference is less than ϵ_T, and a and b fail to be distinguished in tastiness.

One shouldn't be misled by the numerical values in the formalism. A relation $\succeq$ is a semiorder only if *there is* a function f and positive number ϵ such that $x \succeq y$ iff $f(x) \geq f(y) - \epsilon$. As noted in §2.1, degrees needn't be isomorphic to numbers, and properties of ADJ-ness needn't be quantifiable. Talk of the numerical relation between $f_A(adj(a))$ and $f_A(adj(b))$ is compatible with a and b being as ADJ as one another, imperceptibly different in ADJ-ness, or even incomparable.

The relation $\succ_A$ represents pairs of items that count as relevantly distinguishable in the context. The notion of distinguishability here is specific to matters of ADJ-ness. Being discriminable in some respect doesn't imply being "distinguishable," in the sense of being related by $\succ_A$. Conversely, the fact that $adj(a) \sim_A adj(b)$ doesn't imply that a and b are indiscriminable, either in general or in properties relevant to determining how ADJ they are. A supertaster might be able to discriminate between adjacent coffee cups in their quantity of sugar or quality of sweetness (§2.3). To say that the cups' degrees of tastiness are related by $\sim_T$ is to say that any such difference doesn't distinguish them in tastiness. The act of saving 2 strangers at the expense of your dear friend might be discriminable in utility from the act of saving 3 strangers at the expense of your dear friend. Such a difference may or may not determine a relevant distinction in moral value.

3.3 Comparative sorites revisited

Let's apply the semantics to the comparative sorites arguments from §§1–2. To fix ideas I focus on the alternative versions of the example with 'tasty' in (2) and (6). Start with (6), reproduced below, again where x_s is an ordinary sweetened cup of coffee and x_n is a cup with n micrograms of sugar. Truth conditions for the tolerance premise (P2′) and PI-transitivity premise (P3) are in (18)–(19).

(6) (P1) x_s is tastier than x_1.
(P2′) For all n, x_n is as tasty as x_{n+1}.

(P3) For all a, b, c, if a is tastier than b, and b is as tasty as c, then a is tastier than c.

(C) $\therefore$ For all n, x_s is tastier than x_n.

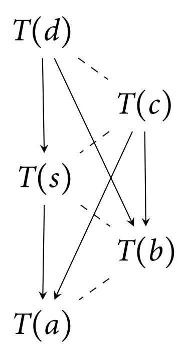
(18) (P2′) is true iff $\forall n$: $T(x_n) \sim_T T(x_{n+1})$

(19) (P3) is true iff $\forall a \forall b \forall c$: $\big(T(a) \succ_T T(b) \wedge T(b) \sim_T T(c)\big) \to T(a) \succ_T T(c)$

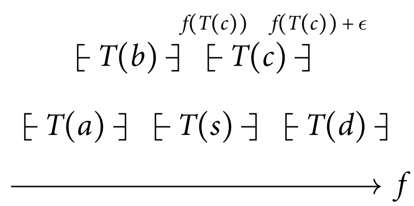
The tolerance premise (P2′) is true according to (18) in the given scenario. The difference in tastiness between any pair of adjacent cups falls below the threshold ϵ_T of distinguishability. Discriminable though they might be, one cup is as good as the next given your preferences. However, (P3) is no longer semantically validated. Semiorders don't in general satisfy PI-transitivity. The subset $\overrightarrow{x \sim y \sim z}$ from (15) is a simple countermodel: $x \succ z$ and $z \sim y$ but $x \nsucc y$, indeed $x \sim y$; hence $(\succ \cdot \sim) \nsubseteq \succ$. The falsity of the conclusion (C) is compatible with the truth of (P1)–(P2′).

Consider a toy example. Let a, b, c, d and s ($=x_s$) be five coffee cups, where s has 1 teaspoon of sugar in it, a has 0.6 teaspoons, b has 0.8 teaspoons, c has 1.2 teaspoons, and d has 1.4 teaspoons. Suppose f_T maps each cup's degree of tastiness to the number of teaspoons of sugar in it, e.g. $f_T(T(s)) = 1$, with $\epsilon_T = 0.3$. Visually:

(20) Graph representation:



Interval representation:



Numerical representation:

	f
$T(a)$	0.6
$T(b)$	0.8
$T(s)$	1.0
$T(c)$	1.2
$T(d)$	1.4
ϵ =	0.3

(P1) is true: $T(s) \succ_T T(a)$; s is tastier than a. And (P2′) is true: $T(a) \sim_T T(b) \wedge T(b) \sim_T T(s) \wedge T(s) \sim_T T(c) \wedge T(c) \sim_T T(d)$; adjacent cups aren't distinguished in tastiness. Yet the intransitivity of $\sim_T$ invalidates (P3), as discussed above, and the conclusion (C) is false: s isn't tastier than itself, b, c, or d, e.g. $T(s) \nsucc_T T(s)$.

The semantics also avoids validating the inductive premises (P2) in (1)–(3) in the given contexts. Truth conditions for (P2) in (2), reproduced below, are in (21).

(2) (P1) x_1 is not tastier than x_s.
(P2) For all n, if x_n is not tastier than x_s, then x_{n+1} is not tastier than x_s.
(C) $\therefore$ For all n, x_n is not tastier than x_s.

(21) (P2) is true iff $\forall n$: $T(x_n) \not\succ_T T(x_s) \to T(x_{n+1}) \not\succ_T T(x_s)$

The counterinstance to (P2) occurs at the cup x_i, $i = \min\{k: T(x_k) \succ_T T(x_s)\} - 1$. In our toy example in (20), cup c is not tastier than s ($=x_s$); the difference in sweetness is insufficient to make one tastier than the other, i.e. $T(c) \sim_T T(s)$. Not so with d. The extra sweetness from the additional 0.4 teaspoons of sugar is significant, as far as your preferences are concerned. d is tastier than s, i.e. $T(d) \succ_T T(s)$, (P2) is false, and (C) doesn't follow.

As Fara (2000) emphasizes, an overall account of the sorites must do more than simply predict that the inductive premise is not true. For instance, if the inductive premise is not true, why do we find it plausible? What should we say about the seemingly predicted "sharp boundary" between (e.g.) cups that are not tastier than x_s and cups that are? Several directions for approaching such questions in the present framework are as follows.

First, crucially, the semantics captures the truth of tolerance claims such as (P2′) without verifying inductive premises such as (P2).

(22) *Tolerance* (P2′): ✓
$\forall n$: $T(x_n) \sim_T T(x_{n+1})$

(23) *Inductive premise* (P2): ✗
$\forall n$: $T(x_n) \not\succ_T T(x_s) \to T(x_{n+1}) \not\succ_T T(x_s)$

Unlike the traditional semantics in §2, the present semiorder-based semantics avoids treating the tolerance claim as implying the inductive premise. The key ingredient is the possible intransitivity of the non-distinguishability relation $\sim$. Scenarios in which $x \sim y \sim z$ invalidate PI-transitivity and falsify (P2). Adjacent items in the series are not relevantly distinguishable in ADJ-ness in the context. In (2)/(6), any cup x_n is as tasty as the next, i.e. $T(x_n) \sim_T T(x_{n+1})$. Paradoxical conclusions needn't follow.

Second, the formal semantics is compatible with alternative philosophical theories of vagueness (e.g., epistemicism, contextualism, supervaluationism). Ideas from such theories can be imported in the treatment of the distinguishability threshold. Let $\delta_A = \min\{k: a_k \succ_A a_n\} - n$ be the "size" of the distinguishability threshold, for every degree of ADJ-ness a_n. (Assume a constant threshold, for simplicity.) On an epistemicist theory (Sorensen 1988, Williamson 1994), facts about competent use across contexts would determine a specific value of δ_A. There would be a context-invariant counterinstance to inductive premises such as (P2) in (1)–(3). The apparent fuzziness in the boundary between (e.g.) coffee cups that are tastier than x_s and those that are not can be diagnosed as uncertainty about the metasemantic facts determining what precise language is being spoken. Alternatively, on a broadly contextualist line, the distinguishability threshold may be treated as a contextual parameter, with different contexts determining different levels of distinguishability. For the maximally discriminating and opinionated among us, context determines a semiorder $\succeq_{A,c}$ such that $\delta_{A,c} = 1$. Every difference in properties determining ADJ-ness affects how ADJ things are. Otherwise, context determines $\delta_{A,c} > 1$ and a comparative sorites is off and running. Even if the compositional semantics assumes a particular representation of context c, there may be a range of scales and values $\delta_{A,c}$ compatible with speakers' interests (cf. Fara 2000), psychological states or verbal dispositions (cf. Raffman 1994, 1996), or discourse moves (cf. Kamp 1981, Soames 1999, Shapiro 2006, Silk 2016, 2021). We may not be able to point to any instance of (P2) we reject, or any instance of the sharp boundaries claim we accept.

We began in §1 by noting the common assumption that comparatives are not vague. We have seen that this assumption is false. Yet, if comparatives can be vague, whence the traditional wisdom that comparatives with "prototypical relative adjectives" (McNally 2011: 163), such as 'tall' in (4), are not?

(4) Alice is taller than Bert.

Vagueness of positive-form predications 'x is ADJ' turns on a fuzziness in how ADJ x must be to count as ADJ. The vagueness of comparatives such as those in §1 turns on a fuzziness in how much of some underlying property makes for a difference in ADJ-ness — how much sweetness makes for a difference in how tasty the coffee is, how much brightness makes for a difference in how pretty the photo is, and so on. It is hard to imagine a situation in which not every difference in height affects how tall something is. Uses of adjectives 'ADJ' in contexts of maximal distinguishability are a limiting case in which the image of the measure function adj, $D' \subseteq D$, is totally ordered by $\succeq_A$ and the semiorder $\langle D', \succeq_A \rangle$ is a linear order.[10] The tolerance claim that $adj(x_n) \sim_A adj(x_{n+1})$, for all n, is false. The comparative sorites doesn't arise.

The semantics developed in this section uses a traditionally defined semiorder, a relation that is "crisp" and complete. There is a precise boundary between cups x_j such that $T(x_j) \sim_T T(x_s)$ and cups x_k such that $T(x_k) \succ_T T(x_s)$; and all elements

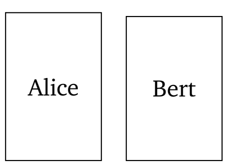
[10]A linear order is a complete, transitive, antisymmetric binary relation; see n. 9. Hereafter I will ignore the possible context-dependence of $\succeq_A$ described above.

are comparable.[11] Both features are inessential to the general framework. Cases with incomparabilities ("incommensurability") between items, such as from incomparable dimensions of application, may call for partial semiorders. The semantics could also be developed with a variant type of threshold structure. For instance, one could use a "pseudo-order," which introduces an intermediate ("hesitation") zone in the transition between non-distinguishability $\sim$ and distinguishability $\succ$ (compare higher-order vagueness). Or, for fans of a many-valued approach to vagueness, a "fuzzy" ("valued") semiorder could be substituted. Formal properties of such alternative threshold structures have been extensively studied (see Moretti et al. 2016: §§5–6 and references therein).

3.4 Comparisons

Several authors have appealed to semiorders in accounts of vagueness phenomena with predicative uses and the positive form (Luce 1956, Halpern 2008, van Rooij 2011a,b, Cobreros et al. 2012). The only precedent that I am aware of for invoking semiorders in an account of comparatives is van Rooij's (2011a) delineation semantics for "implicit" comparatives (Kennedy 2007a, 2011) — sentences like 'x is ADJ compared to y' in which a comparison is made using the positive form. The truth of implicit comparatives requires that the difference between the items be significant, as reflected in (24) (cf. van Rooij 2011a: 65–66).

(24)



a. Alice is taller than Bert. (*true*)
b. Alice is tall compared to Bert. (*false*)

van Rooij (2011a) uses semiorders to capture this "significantly ADJ-er than" relation in the interpretation of comparative uses of the positive form such as (24b). "[S]tandard explicit comparatives like" (24a) with 'taller' are analyzed, as is common, via weak orders (2011a: 65; cf. §2.2). The semantics in this section, in contrast, invokes semiorders in the scale structure and allows for vagueness phenomena with both positive and comparative forms. There may still be semantic differences between implicit and explicit comparatives. The difference in height that renders (24a) true needn't be "significant" so as to verify (24b).

[11] Reflexivity and the Ferrers property (§3.1) entail completeness, i.e. $x \geq y \vee y \geq x$, for all $x, y \in S$.

4 Recap

Despite the vast literature on linguistic vagueness, vagueness phenomena with comparatives have received little attention. Narrow focus on adjectives such as 'tall' has led many theorists to assume that the comparative form cannot be vague. Sorites examples such as (1)–(3), (6) illustrate that this assumption is false. Vagueness phenomena with comparatives '*x* is ADJ-er than *y*' can arise in situations of fuzziness regarding how much of some ADJ-determining property makes for a difference in ADJ-ness. Vagueness can be associated not only with how ADJ something must be to count as ADJ, but with how ADJ things are. The latter sort of vagueness cannot be assimilated to indiscriminability or fuzziness in relevant dimensions or measurement procedures.

Traditional semantics for gradation validate transitivities that yield paradoxical conclusions in comparative sorites arguments (§2). §3 developed a revised semantics with semiorders, a type of well studied threshold structure, that avoids this problem. The semantics captures the truth of tolerance claims expressing that adjacent items in a sorites series are not relevantly distinguishable in ADJ-ness. And the semantics avoids treating the tolerance claim as implying the inductive premise in a three-step sorites argument. How exactly one responds to the paradox and other vagueness phenomena will depend on one's preferred theory of vagueness. The formal semantics can be adapted accordingly.

References

Aleskerov, Fuad, Denis Bouyssou & Bernard Monjardet. 2007. *Utility maximization, choice and preference*. Berlin: Springer, 2nd edn.

Bale, Alan. 2008. A universal scale of comparison. *Linguistics and Philosophy* 31: 1–55.

Barker, Chris. 2002. The dynamics of vagueness. *Linguistics and Philosophy* 25: 1–36.

Bartsch, Renate & Theo Vennemann. 1973. *Semantic structures: A study in the relation between syntax and semantics*. Frankfurt: Athäenum.

van Benthem, Johan. 1982. Later than late: On the logical origin of the temporal order. *Pacific Philosophical Quarterly* 63: 193–203.

Bhatt, Rajesh & Roumyana Pancheva. 2007. Degree quantifiers, position of merger effects with their restrictors, and conservativity. In Chris Barker & Pauline Jacobson (eds.), *Direct compositionality*, 306–335. New York: Oxford University Press.

Bochnak, Michael Ryan. 2013. *Cross-linguistic variation in the semantics of comparatives*. University of Chicago, Ph.D. thesis.

Bouyssou, Denis & Marc Pirlot. 2021a. Unit representation of semiorders I: Countable sets. *Journal of Mathematical Psychology* 103(102566): 1–21.

Bouyssou, Denis & Marc Pirlot. 2021b. Unit representation of semiorders II: The general case. *Journal of Mathematical Psychology* 103(102568): 1–19.

Burnett, Heather Susan. 2012. *The grammar of tolerance: On vagueness, context-sensitivity, and the origin of scale structure*. UCLA, Ph.D. thesis.

Cobreros, Pablo, Paul Égré, David Ripley & Robert van Rooij. 2012. Tolerant, classical, strict. *Journal of Philosophical Logic* 41: 347–385.

Cooper, Neil. 1995. Paradox lost: Understanding vague predicates. *International Journal of Philosophical Studies* 3: 244–269.

Cresswell, M.J. 1977. The semantics of degree. In Barbara Partee (ed.), *Montague grammar*, 261–292. New York: Academic Press.

Égré, Paul & Nathan Klinedinst. 2011a. Introduction: Vagueness and language use. In Égré & Klinedinst (2011b), 1–21.

Égré, Paul & Nathan Klinedinst (eds.). 2011b. *Vagueness and language use*. New York: Palgrave Macmillan.

Endicott, Timothy. 2000. *Vagueness in law*. Oxford: Oxford University Press.

Fara, Delia Graff. 2000. Shifting sands: An interest-relative theory of vagueness. *Philosophical Topics* 28: 45–81.

Fine, Kit. 1975. Vagueness, truth, and logic. *Synthese* 30: 265–300.

Fishburn, Peter C. 1970. Intransitive indifference in preference theory: A survey. *Operations Research* 18: 207–228.

Fishburn, Peter C. 1973. Interval representations for interval orders and semiorders. *Journal of Mathematical Psychology* 10: 91–105.

Fishburn, Peter C. 1985. *Interval orders and interval graphs: A study of partially ordered sets*. New York: Wiley.

Halpern, Joseph Y. 2008. Intransitivity and vagueness. *The Review of Symbolic Logic* 1: 530–547.

Heim, Irene. 2001. Degree operators and scope. In Caroline Féry & Wolfgang Sternefeld (eds.), *Audiatur vox sapientiae: A festschrift for Arnim von Stechow*, 214–239. Berlin: Akademie Verlag.

Kamp, Hans. 1981. The paradox of the heap. In Uwe Mönnich (ed.), *Aspects of philosophical logic*, 225–277. Dordrecht: Reidel.

Keefe, Rosanna. 2000. *Theories of vagueness*. Cambridge: Cambridge University Press.

Kennedy, Christopher. 1999. *Projective the adjective: The syntax and semantics of gradability and comparison*. New York: Garland Publishing.

Kennedy, Christopher. 2007a. Modes of comparison. In Malcolm Elliott, James Kirby, Osamu Sawada, Eleni Staraki & Suwon Yoon (eds.), *Proceedings of the Chicago Linguistic Society 43*, 141–165. Chicago: CLS.

Kennedy, Christopher. 2007b. Vagueness and grammar: The semantics of relative and absolute gradable adjectives. *Linguistics and Philosophy* 30: 1–45.

Kennedy, Christopher. 2011. Vagueness and comparison. In Égré & Klinedinst (2011b), 73–97.

Kennedy, Christopher. 2013. Two sources of subjectivity: Qualitative assessment and dimensional uncertainty. *Inquiry* 56: 258–277.

Klein, Ewan. 1980. A semantics for positive and comparative adjectives. *Linguistics and Philosophy* 4: 1–45.

Klein, Ewan. 1991. Comparatives. In Arnim von Stechow & Dieter Wunderlich (eds.), *Semantics: An international handbook of contemporary research*, 673–691. New York: de Gruyter.

Lassiter, Daniel. 2015. Adjectival modification and gradation. In Shalom Lappin & Chris Fox (eds.), *The handbook of contemporary semantic theory*, 143–167. Oxford: Wiley-Blackwell, 2nd edn.

Lewis, David. 1970. General semantics. *Synthese* 22: 18–67.

Luce, R. Duncan. 1956. Semiorders and a theory of utility discrimination. *Econometrica* 24: 178–191.

McNally, Louise. 2011. The relative role of property type and scale structure in explaining the behavior of gradable adjectives. In Nouwen et al. (2011), 151–168.

Moretti, Stefano, Meltem Öztürk & Alexis Tsoukiàs. 2016. Preference modelling. In Salvatore Greco, Matthias Ehrgott & José Rui Figueira (eds.), *Multiple criteria decision analysis: State of the art surveys*, 43–95. New York: Springer, 2nd edn.

Morzycki, Marcin. 2015. *Modification*. Cambridge: Cambridge University Press.

Nouwen, Rick, Robert van Rooij, Uli Sauerland & Hans-Christian Schmitz (eds.). 2011. *Vagueness in communication*. Berlin: Springer-Verlag.

Pirlot, Marc & Philippe Vincke. 1997. *Semiorders: Properties, representations, applications*. Dordrecht: Kluwer.

Raffman, Diana. 1994. Vagueness without paradox. *Philosophical Review* 103: 41–74.

Raffman, Diana. 1996. Vagueness and context relativity. *Philosophical Studies* 81: 175–192.

Raffman, Diana. 2014. *Unruly words: A study of vague language*. Oxford: Oxford University Press.

Rett, Jessica. 2008. *Degree modification in natural language*. Rutgers University, Ph.D. thesis.

van Rooij, Robert. 2011a. Implicit versus explicit comparatives. In Égré & Klinedinst (2011b), 51–72.

van Rooij, Robert. 2011b. Vagueness, tolerance and non-transitive entailment. In Petr Cintula, Christian G. Fermüller, Godo Lluís & Petr Hájek (eds.), *Understanding vagueness: Logical, philosophical and linguistic perspectives*, 205–222. London: College Publications.

Russell, Bertrand. 1923. Vagueness. *Australasian Journal of Psychology and Philosophy* 1: 84–92.

Sassoon, Galit W. 2011. The inhabitants of vagueness models. In Égré & Klinedinst (2011b), 98–118.

Sassoon, Galit W. 2013. *Vagueness, gradability and typicality: The interpretation of adjectives and nouns*. Leiden: Brill.

Scott, Dana & Patrick Suppes. 1958. Foundational aspects of theories of measurement. *Journal of Symbolic Logic* 23: 113–128.

Shapiro, Stewart. 2006. *Vagueness in context*. New York: Oxford University Press.

Silk, Alex. 2016. *Discourse Contextualism: A framework for contextualist semantics and pragmatics*. Oxford: Oxford University Press.

Silk, Alex. 2021. Evaluational adjectives. *Philosophy and Phenomenological Research* 102: 127–161.

Soames, Scott. 1999. *Understanding truth*. Oxford: Oxford University Press.

Sorensen, Roy. 1988. *Blindspots*. Oxford: Clarendon.

von Stechow, Arnim. 1984. Comparing semantic theories of comparison. *Journal of Semantics* 3: 1–77.

Suppes, Patrick, David H. Krantz, R. Duncan Luce & Amos Tversky. 1989. *Foundations of measurement, Vol. 2: Geometrical, threshold, and probabilistic representations*. New York: Academic Press.

Williamson, Timothy. 1994. *Vagueness*. London: Routledge.

Wright, Crispin. 1975. On the coherence of vague predicates. *Synthese* 30: 325–365.

Wright, Crispin. 1987. Further reflections on the Sorites paradox. *Philosophical Topics* 15: 227–290.