1 Introduction

Fuel cell systems (FCS) can be used as a primary power source for both automotive applications and stationary power applications. Using an energy buffer together with the FCS can provide benefits such as reduced hydrogen consumption and assistance during transient loading. The hybridization levels for reduced hydrogen consumption of an automotive system are addressed in [1], where it is shown that when not considering regenerative braking, the hybridization of automotive fuel cell systems will not always improve fuel economy. The hybridization levels for protecting the FCS from abrupt load variations that can cause reactant starvation are clarified here. Long reactant starvation periods can lead to decreased performance and permanent damage of the fuel cell membrane. Judicious hybridization can thus extend the lifetime of FCS, which is currently one of the technical hurdles associated with FCS.

Hybridization can provide assistance during FCS transient loading and thus reduce oxygen starvation in both automotive and stationary fuel cell applications. In general, the hydrogen at the anode is provided by a pressurized hydrogen storage and the oxygen at the cathode is provided by a compressor. We will only consider oxygen starvation in this study and assume an instantaneous hydrogen supply.

To prevent oxygen starvation, an excess oxygen supply is desired. However, an increase in oxygen supply increases the parasitic losses associated with increased compressor output. In this paper, the effects of different lead-acid battery sizes in a fuel cell hybrid electric vehicle have been analyzed when the FCS oxygen excess ratio (OER) needs to meet certain constraints. Note that particle matter and smoke in Diesel engine exhaust forms during transient loading, and thus our approach, could be applied to an ICE Diesel-hybrid vehicle. In this paper, the power split is first achieved through dynamic programming (DP) [2] optimization. The DP results represent the best-possible sequence of commands, which can only be achieved by knowing a priori the course that will be driven. The power split derived through DP is a-causal. As it is shown later, the optimum power split policy often dictates an increase in FCS power before an increase in the requested power. This precompensation helps reduce excursions in OER and battery state of charge (SOC), but cannot be realized when the future power demand is not known, as in a typical driving scenario.

In the last part of this paper, the power split is controlled by a simple nonlinear and causal controller. The results associated with the FCS performance and the battery sizing for the two control schemes are compared and analyzed.

When hybridizing a power train, it is challenging to size the battery because the drive cycle [3,4], the control policy [5], and the hardware architecture [6] affect the optimal size. When increasing the battery size in a hybrid electric vehicles, the total vehicle weight increases, which affects the fuel consumption. In stationary fuel cell applications, the weight is not as important as in automotive applications; thus, if the same approach is used to size an energy buffer, additional performance variables, such as cost or volume, have to be included in the DP optimization.

When making the decision to include a battery or another energy buffer in the power train, it is important to consider the actual costs of the battery and the added complexity associated with hybridization [3,7]. The prices on fuel cells and batteries vary with time; we will therefore only focus on determining the effects of hybridization on hydrogen consumption and stack oxygen excess ratio. We do not know how detrimental to the FCS life are periods of low excess oxygen ratio; thus, we choose an arbitrary function to penalize deviations of OER.

Section 2 describes the fuel cell hybrid electric vehicle (FCHEV) model. Section 3 shows the dynamic programming method and results. In Sec. 4, a simple causal controller is presented and a comparison to the dynamic programming results is presented. Finally, in Sec. 5, a discussion and some conclusions from the results are discussed as well as possible future work.
Fig. 1 Signal flow overview of the two methods used to control the power split in the FCHEV. Top figure (a) shows the signal flow during the dynamic programming optimizations. Bottom figure (b) shows an overview of the proposed controller and the signal flows between the controller and the FCHEV model.

2 Method

To investigate how battery size affects the optimum FCS buffering during transient loading, we will first use deterministic DP and then a simple causal and nonpredictive controller. The DP methodology weights hydrogen consumption, excursions from the nominal fuel cell oxygen concentration and excursions from the nominal battery state of charge as shown in Fig. 1(a). The simple causal nonpredictive controller is used to control the battery state of charge as shown in Fig. 1.

2.1 FCHEV Model. The FCHEV model, similar to [1], is separated into three components: fuel cell system, vehicle, and battery pack. Figure 1(a) shows the three components and how they interact with each other during the DP optimizations, and Fig. 1(b) shows how the three components interact when using the proposed controller. The FCHEV model is a simplified version of the detailed model in [8]. When using DP to solve optimal control problems, it is crucial to minimize the number of states and the computation time of the model due to the computational complexity of the DP algorithm. This is done by approximating fast dynamics as instantaneous and employing nonlinear static maps to model their associated steady state behavior. Forward Euler approximation, with a sampling interval of 1 s, is used to discretize the continuous in time state equations before using DP and the proposed controller. The parameters in the model are shown in Table 1.

2.1.1 Fuel Cell System. The FCS is the primary energy source in the vehicle that supplies electric energy to the voltage bus in the vehicle. An important FCS performance variable is the oxygen excess ratio (OER), which is the mass airflow rate supplied to the cathode divided with the mass airflow rate needed for the oxygen reaction associated with the current drawn from the fuel cell. Maintaining high OER helps in reducing the possibility of oxygen starvation in the fuel cell stack. Steady-state regulation of OER is shown to be achieved by a feedforward compressor controller [9]. During fast changes in current drawn, it is not possible to maintain the desired OER value due to the manifold filling and compressor inertial dynamics. It is hence important to develop an optimal power-splitting policy based on its effects on OER excursions. To penalize OER excursions in the DP cost function, we extend the model presented in [1] by introducing an extra state that allows through the map in Fig. 2 to access the worst OER, \( \lambda_{OER} \), during changes in reference current \( I_{ref} \): \( \lambda_{OER} = g(I_{ref}, \Delta I_{ref}) \), where \( \Delta I_{ref}(k) = I_{ref}(k) - I_{ref}(k-1) \). The map \( g \) is calculated using the original FCS model [8], which includes (i) a feedforward compressor map that maintains a steady-state OER at 2, and (ii) the dynamics of compressor and manifolds. With the map \( g \) and the additional state, we manage to accurately capture OER excursions without the three extra states of the FCS model in [8].

Two examples of the worst OER value during step changes in the reference current are shown in Fig. 3. A step increase of 10 A from 110 A to 120 A causes OER to drop to 1.77. The worst OER value of 1.77 is thus data point (a) in Fig. 2. A step of 10 A from 10 A to 20 A causes OER to drop momentarily down to 1.0 as shown in Fig. 3 and summarized with data point (b) in Fig. 2.

The rest of the FCS model is the same as the one in [1]. The total energy in the used fuel during the cycle \( E_{H_2} \) is

\[
E_{H_2} = Q_{H_2}^H \int_0^T W_{H_2} dt \ [J]
\]  

(1)

The hydrogen consumption \( W_{H_2} \) is a function of the current drawn from the FC stack, \( I_{ref} \), which, in turn, is a function of the net

![Hydrogen consumption](image)

**Fig. 2** Oxygen excess ratio map used to calculate the OER in the fuel cell system when changing the reference current. The examples (a) and (b) shown in Fig. 3 are also marked.
The acceleration power demand, \( P_{\text{acc}} \), where battery output/input power drop, and a step of 10 A from 100 A (b) and the resulting OER drop.

Current out of the FCS \( I_{\text{fcs}} \) dictated by the controller. The total energy out from the FCS \( E_{\text{fcs}} \) is

\[
E_{\text{fcs}} = \int_0^t (P_{\text{fcs}} - P_{\text{aux}}) dt \quad [\text{J}]
\]

where \( P_{\text{aux}} \) is a fixed power demand from all the other FCS auxiliary devices and \( P_{\text{fcs}} \) is the FCS output power, which is a function of the net FCS current as in [1]. The maximum FCS power is \( P_{\text{fcs}}^{\text{max}} = 54 \text{ kW} \) and occurs at \( I_{\text{ref}} = 248 \text{ A} \). Further increase in the current drawn from the FCS decreases the net FCS power so the FCS reference current is limited to \( I_{\text{ref}} = I_{\text{max}}^{\text{fcs}} = 248 \text{ A} \) throughout this study.

The FCS efficiency is

\[
\eta_{\text{fcs}} = \frac{P_{\text{fcs}} - P_{\text{aux}}}{Q_{\text{HHV}}^H W_{\text{H}_2}}
\]

where \( Q_{\text{HHV}}^H \) is the energy content of hydrogen (using higher heating value). The cycle average FCS efficiency is

\[
\bar{\eta}_{\text{fcs}} = \frac{E_{\text{fcs}}}{E_{\text{H}_2}}
\]

The parameters in the FCS model are shown in Table 1.

\[\text{Table 2 Drive-cycle characteristics and the power characteristics when } \eta_{\text{bat}} = 10 \text{ and } m = 1451 \text{ kg}\]

<table>
<thead>
<tr>
<th>Cycle</th>
<th>NYCC</th>
<th>FTP-72</th>
<th>SFTP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average power</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Acc. (kW)</td>
<td>4.9</td>
<td>7.2</td>
<td>21.5</td>
</tr>
<tr>
<td>Brk. (kW)</td>
<td>4.7</td>
<td>7.1</td>
<td>17.3</td>
</tr>
<tr>
<td>Max power</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Acc. (kW)</td>
<td>25.7</td>
<td>31.8</td>
<td>80.1</td>
</tr>
<tr>
<td>Brk. (kW)</td>
<td>21.6</td>
<td>23.4</td>
<td>53.8</td>
</tr>
<tr>
<td>Top speed (km/h)</td>
<td>44.6</td>
<td>91.3</td>
<td>129.2</td>
</tr>
<tr>
<td>Duration (s)</td>
<td>599</td>
<td>1370</td>
<td>601</td>
</tr>
<tr>
<td>Distance (m)</td>
<td>1899</td>
<td>11990</td>
<td>12888</td>
</tr>
<tr>
<td>( E_{\text{O2}} ) (MJ)</td>
<td>1.024</td>
<td>5.748</td>
<td>8.980</td>
</tr>
</tbody>
</table>

2.1.2 Vehicle. The vehicle mass 1384 kg includes the FCS and hydrogen storage and excludes the mass of the battery pack. The power demand \( P_{\text{dem}} \) is

\[
P_{\text{dem}}(m,t) = v(t) \left[ m \dot{u}(t) + K_{\text{mg}} + P_{\text{aux}} \frac{C_p A}{2} u^2(t) \right] \quad [\text{W}]
\]

The acceleration power demand, \( P_{\text{acc}}(t) \), is the positive part of the power demand, \( P_{\text{dem}}(5) \), provided by the fuel cell system and the battery pack. The deceleration power demand is assumed to be absorbed by nonregenerative braking.

2.1.3 Battery Pack. The battery pack model is the same with the one used in [1] and is based on an ADVISOR model [10]. The battery output/input power \( P_{\text{bat}} \) is the remaining power to meet the drive-cycle power demand

\[
P_{\text{bat}}(t) = P_{\text{acc}}(t) - (P_{\text{fcs}}(t) - P_{\text{aux}}) \quad [\text{W}]
\]

where \( P_{\text{acc}} \) is the acceleration power demand and \( (P_{\text{fcs}} - P_{\text{aux}}) \) is the FCS net output power. The battery current \( I_{\text{bat}} \) is a function of the battery’s state of charge, as in [1]. The battery’s state of charge (SoC) is calculated using

\[
\frac{d}{dt}[\text{SoC}(t)] = - \frac{I_{\text{bat}}(t) \eta_{\text{bat}}[I_{\text{bat}}(t)]}{3600 \eta_{\text{bat}}} \quad (7)
\]

where \( \eta_{\text{bat}} \) is the battery charging efficiency [1]. The input power, i.e., charging current, to the battery has not been limited, and all the charging power from the FCS and the regenerative braking is assumed to be absorbed by the battery.

2.2 Drive Cycles. The power demand from the drive cycles is calculated backward using the discretized version of the vehicle model (5). The three drive cycles are New York City Cycle (NYCC), which represents low speed and mild driving; federal test procedure—72 cycle (FTP-72), which represents both low-speed city driving and moderate highway driving; and supplemental federal test procedure cycle (SFTP), which represents aggressive high-speed driving. The drive cycles, and their power characteristics for a vehicle with ten battery modules and a mass of 1451 kg, are summarized in Table 2.

3 Dynamic Programming Optimizations

This section shows the DP method and the results from the DP optimizations. The general signal flow during the optimizations is shown in Fig. 1(a), where \( v \) is the speed given by the drive cycles and \( I_{\text{ref}} \) is the input reference current to the FCS. The three performance variables, the battery’s state of charge SoC, the hydrogen consumption in the FCS \( W_{\text{H}_2} \), and the OER in the FCS \( \lambda_{\text{O}_2} \) are used to calculate the cost function for the DP optimization. We will investigate, as in [1], the energy losses in the different components and how they change with the battery size.

3.1 Dynamic Programming Method. The main DP equations, equivalent to [1], are shown in Appendix. To incorporate the OER dynamics in the optimization, however, an additional state is introduced compared to [1]. Moreover, the cost function, \( J \), has to be redefined as

\[
J = (\alpha \Delta \text{SoC})^2 + \beta W_{\text{H}_2} + (\gamma \Delta \lambda_{\text{O}_2})^2
\]

\[
\Delta \text{SoC} = \text{SoC} - \text{SoC}_{\text{ref}}
\]

\[
\Delta \lambda_{\text{O}_2} = \begin{cases} \lambda_{\text{lim}}^{\text{O}_2} - \lambda_{\text{O}_2}, & \lambda_{\text{O}_2} < \lambda_{\text{lim}}^{\text{O}_2} \\ 0, & \lambda_{\text{O}_2} \geq \lambda_{\text{lim}}^{\text{O}_2} \end{cases}
\]

where \( \Delta \text{SoC} \) is the deviation of the battery’s state of charge from the reference value \( \text{SoC}_{\text{ref}} \), \( W_{\text{H}_2} \) is the hydrogen consumption in the FCS, \( \Delta \lambda_{\text{O}_2} \) is the deviation of the minimum OER (when changing the reference current) below a limiting value \( \lambda_{\text{lim}}^{\text{O}_2} \), and \( \{\alpha, \beta, \gamma\} \) are weights. FCS or battery deterioration are not in-
3.2.1 Fuel Cell System. The average FCS efficiency (4) is shown in the left column of Fig. 4. When increasing the battery size the FCS efficiency increases. This increase is caused, as in [1], by decoupling the acceleration power demand, \( P_{acc} \), from the FCS output power, \( P_{fcs} \). The right column of Fig. 4 shows the distribution of the FCS reference current \( I_{ref} \) for different sizes together with the FCS efficiency curve \( \eta_{fcs} \). The changes of these distributions are similar to those in [1]. When increasing the battery size, the DP shifts the inefficient current levels toward higher efficiencies. The average FCS efficiencies achieved now are larger than the ones observed when OER was not penalized [1] because the peak reference current and very low reference current levels that typically cause large OER excursions are now rare. The peak reference current now shift to lower values with high FCS efficiencies as shown in Fig. 4 (right column).

The resulting OER distributions for the different drive cycles and battery sizes are shown in Fig. 5. For example, with a battery size of five modules in the medium cycle (FTP-72) the OER is higher than 1.8, 70% of the time, and higher than 1.68, 100% of the time. Furthermore, with a battery size of 19 modules in the mild cycle (NYCC) the OER is higher than 1.7, 100% of the time and higher than 1.95, 90% of the time.

3.2.2 Battery. The electrical energy expended in the battery is

\[
E_{\text{loss}}^{\text{btt}} = \left| \int_0^T P_{\text{btt}} \, dt \right| \quad [J]
\]

where \( P_{\text{btt}} \) is the battery power.\(^2\) The hydrogen energy required to produce the electric energy expended in the battery is \( E_{\text{loss}}^{\text{btt}} / \eta_{fcs} \), where \( \eta_{fcs} \) is the cycle average FCS efficiency. The energy expended in the battery, \( E_{\text{loss}}^{\text{btt}} \), together with the hydrogen equivalent energy, \( E_{\text{loss}}^{\text{hyd}} / \eta_{fcs} \), are shown in the left column of Fig. 6.

For the mild and medium cycles the expended battery energy decreases rapidly at first but then levels out for larger battery sizes. For the aggressive cycle, the expended battery energy is also decreasing with increasing battery size. This is because the DP is forced, through the added cost in (8), to use the battery to maintain the OER at high levels. This is hard for small batteries and will therefore result in larger expended energy in the battery.

3.2.3 Added Mass. The energy loss due to the added mass, when increasing the battery size, affects the performance of the vehicle. We define the energy loss due to the added mass as

\[
E_{\text{loss}}^{\text{add}} = E_{\text{acc}}^{\text{acc}} - E_{\text{acc}}^{\text{acc}}[J]
\]

where the total acceleration energy for a vehicle with \( n_{\text{btt}} \) battery modules is

\[^2\text{Note that even though the DP ensures that } \text{SoC}(T) = \text{SoC}(0) \text{ there is always energy expended in the battery } \left( E_{\text{loss}}^{\text{btt}} > 0 \right) \text{ due to losses in the internal resistance and due to the charging efficiency, } \eta_{fcs} \text{ [1].} \]
due to the added mass, and the total acceleration energy for a vehicle without battery pack is

\[ E_{acc}^0 = \int_0^T P_{acc}(m_0 + n_{bat}m_{bat}, t) \, dt \]  

(14)

The parameters \( m_0 \) and \( m_{bat} \) are shown in Table 1. The energy loss due to the added mass \( E_{loss}^m \) and the hydrogen equivalent energy loss \( E_{loss}^m/\eta_{fcs} \) are shown in the right column of Fig. 6. The energy loss \( E_{acc}^0 \) is increasing proportional to the battery size for all three drive cycles. Note that an increase in battery size corresponds to an increase in vehicle weight and net vehicle power because the FCS power size remains fixed.

3.2.4 System. The total hydrogen energy lost during a cycle is defined as the hydrogen energy used during the cycle minus the energy supplied to meet the acceleration power demand for the original vehicle mass, \( m_0 \)

\[ E_{loss} = E_{loss}^m - E_{loss}^{acc} \]  

(15)

The total hydrogen energy lost, \( E_{loss}^{fcs} \), is separated into

\[ E_{loss}^{fcs} = \frac{E_{loss}}{\eta_{fcs}} + \frac{E_{loss}^{bat}}{\eta_{fcs}} + \frac{E_{loss}^{acc}}{\eta_{fcs}} \]  

(16)

where \( E_{loss}^{bat}/\eta_{fcs} \) is the hydrogen equivalent energy expended in the battery, \( E_{loss}^{acc}/\eta_{fcs} \) is the hydrogen equivalent energy loss due to the added mass, and \( E_{loss}^{fcs}/\eta_{fcs} \) is the remaining hydrogen equivalent energy loss in the FCS. The electric energy expended in the battery, \( E_{loss}^{bat} \), is defined in (11), and the electric energy loss due to added weight is defined in (12).

Taking into account the electric energy expended in the battery, \( E_{loss}^{bat} \), defined in (11) and the electric energy loss due to added weight defined in (12), the remaining electric energy loss in the FCS is then calculated using

\[ E_{loss}^{fcs} = \eta_{fcs}E_{loss}^0 - E_{loss}^{bat} - \frac{\Delta m}{m_0} \]  

(17)

similarly to [1].

The total hydrogen energy loss, \( E_{loss}^{fcs} \), is proportional to the gasoline equivalent fuel consumption

\[ C_{gas} = 10^8 \frac{E_{loss}^0 + E_{loss}^{acc}}{Q_{HHV}P_{gas}\int_0^T udT} \frac{1}{100 \text{ km}} \]  

(18)

where \( Q_{HHV} \) is the energy content of gasoline (based on the higher heating value) and \( P_{gas} \) is the gasoline density.

The total hydrogen energy loss, the hydrogen equivalent energy loss due to the added weight, and the hydrogen equivalent energy expended in the battery are shown in Fig. 7. For the mild cycle, the large energy expended in the battery for small battery sizes makes it inefficient to have less battery modules than 10–20. For the medium cycle, the large energy expended in the battery for small battery sizes together with the large energy loss due to the added weight for large batteries introduce an optimal battery size of five to ten modules. In the aggressive cycle, the decrease in expended battery energy is almost equivalent to the energy loss due to added weight. However, there is an optimal battery size of 15 modules.

4 Comparison to Fixed Structure Controller

Since DP is predictive and noncausal, this section introduces a very simple controller to control the state of charge in the battery pack. The resulting controller is used to compare the battery sizing using the predictive optimal DP method to a nonpredictive con-

\[ \text{Note that the gasoline equivalent fuel consumption is measured in \( 1/100 \text{ km} \) and that the US fuel economy, measured in miles per gallon, is } C_{gas,US} = 235.2 \left( \frac{C_{gas}}{100 \text{N}} \right)^{-1} \text{ mpg.} \]
controller. The approach used is a linear proportional controller (PC) controlling an exact linearization of the process. The resulting system then becomes a first-order system with a time constant $\tau$ given by the PC. An overview of the proposed controller is shown in Fig. 1(b).

### 4.1 Proportional Controller With Exact Linearization.

The model can be exactly linearized and controlled by a linear controller. This is done by using the desired change in state of charge as input, the reference current as output, the power demand as a disturbance, and calculating an inverse version of the FCHEV model in the controller. Hence the overall system response, for the battery state of charge, will have the characteristics of a linear system. The battery state of charge is compared to the reference state of charge ($\text{SoC}_{\text{ref}}=0.6$ as discussed above) to form an error that the controller needs to compensate for. The error in the state of charge is then scaled to form the desired change in the state of charge

$$\Delta \text{SoC}_{\text{des}} = \frac{1}{\tau}[\text{SoC}_{\text{ref}} - \text{SoC}(k)]$$

A large value for the constant $\tau$, for example, corresponds to slow rate of battery charging after an initial discharge. The desired battery current $I_{\text{ref}}^\text{des}$ for a desired change in state of charge, $\Delta \text{SoC}_{\text{des}}$, is

$$I_{\text{ref}}^\text{des}(\Delta \text{SoC}_{\text{des}}) = \frac{-3600 \eta_{\text{bat}} \Delta \text{SoC}_{\text{des}}}{\eta_{\text{bat}}(\Delta \text{SoC}_{\text{des}})} [\text{A}]$$

where the battery charging efficiency $\eta_{\text{bat}}$ is

$$\eta_{\text{bat}}(\Delta \text{SoC}_{\text{des}}) = \begin{cases} 1 & \Delta \text{SoC}_{\text{des}} < 0 \\ 0.9 & \Delta \text{SoC}_{\text{des}} > 0 \end{cases}$$

The desired battery output power $P_{\text{bat}}^\text{des}$ is then

$$P_{\text{bat}}^\text{des}(\Delta \text{SoC}_{\text{des}}) = \frac{V_{\text{bat}}^2}{2} - (V_{\text{bat}} - 2P_{\text{bat}}^\text{des}(\Delta \text{SoC}_{\text{des}})R_{\text{int}})^2}{4R_{\text{int}}} [\text{W}]$$

and the desired FCS output power

$$P_{\text{FCS}}^\text{des}(\Delta \text{SoC}_{\text{des}}) = P_{\text{soc}} - P_{\text{bat}}^\text{des}(\Delta \text{SoC}_{\text{des}}) + P_{\text{aux}} [\text{W}]$$

where the power demand, $P_{\text{soc}}$, is known to the controller. The FCS power output map [1], denoted $P_{\text{soc}} = f(I_{\text{ref}})$, is a monotonically increasing function for $I_{\text{ref}} < I_{\text{ref}}^{\text{max}} = 248 \text{ A}$ and hence has the inverse $I_{\text{ref}} = f^{-1}(P_{\text{soc}})$. The exact linearization of the model from the change in SoC, $\Delta \text{SoC}$, to the reference current $I_{\text{ref}}$ is then

$$I_{\text{ref}}^\text{des}(\Delta \text{SoC}_{\text{des}}) = f^{-1}[P_{\text{soc}}^\text{des}(\Delta \text{SoC}_{\text{des}})] [\text{A}]$$

The map in Fig. 2 used to calculate the OER is

$$\lambda_{\text{O}_2}(k) = g[I_{\text{ref}}(k) - I_{\text{ref}}(k-1)]$$

and is monotonically decreasing with increasing $I_{\text{ref}}(k)$ for a given $I_{\text{ref}}(k-1)$. Hence, the function $g$ has the partial inverse

$$I_{\text{ref}}^{\text{max}}(\lambda_{\text{O}_2}) = g^{-1}[-I_{\text{ref}}(k-1), \lambda_{\text{O}_2}^\text{lim}(k)] [\text{A}]$$

where $I_{\text{ref}}^{\text{max}}(\lambda_{\text{O}_2})$ is the maximum reference current that keeps the OER higher than $\lambda_{\text{O}_2}^\text{lim}$ for a given $I_{\text{ref}}(k-1)$. The desired reference current, $I_{\text{ref}}^\text{des}$, in the controller is limited by $\lambda_{\text{O}_2}^\text{lim}

$$I_{\text{ref}}(k) = \min\left\{I_{\text{ref}}^\text{max}^{\lambda_{\text{O}_2}}, I_{\text{ref}}^\text{ref}, I_{\text{ref}}^\text{des}(\lambda_{\text{O}_2}) \right\} [\text{A}]$$

If $I_{\text{ref}}(k) = I_{\text{ref}}^\text{ref}(k)$, i.e., the desired change in the SOC is slow enough that it does not require large FC currents, then the desired change in the battery SOC can be met and the actual battery charging current (19) becomes the closed-loop response,

$$\text{SoC}(k+1) - \text{SoC}(k) = \frac{1}{\tau}[\text{SoC}_{\text{ref}} - \text{SoC}(k)]$$

An example of a step in power demand and the resulting FCS power output, SoC, OER, $P_{\text{soc}}$, and $I_{\text{ref}}$ are shown in Fig. 8. The example shows a vehicle with ten battery modules, a time constant $\tau=30 \text{ s}$, an OER limit of 1.75, and an initial power demand of 10 kW. The step of 20 kW shows that the desired reference current from the controller is limited by $\lambda_{\text{O}_2}^\text{lim}$ but that the SoC is finally regulated to the reference SoC, $\text{SoC}_{\text{ref}}=0.6$.

To compare the proportional controller to the DP optimizations the time constant $\tau$ must be investigated and set to a reasonably good value. Figure 9 shows the hydrogen equivalent total energy...
loss of the vehicle with the PC controlling the SoC with an OER limit, for the three drive cycles, with different time constants and battery sizes. The time constant has been varied between 10 s and 90 s.

The optimal pair of time constant and battery size, in Fig. 9, for each of the three drive cycles differ. A time constant \( \tau = 50 \) s is used to compare the proportional controller to the DP optimizations. Figure 9 shows both the time constant \( \tau \) (horizontal dotted) and the corresponding optimal battery size \( n_{\text{opt}}(\tau) \) (vertical dotted) for each of the cycles.

The longer time constant the more time is required, after the drive cycle’s duration (Table 2), to recharge the battery to the reference state of charge \( \text{SoC}_{\text{ref}} \). The right column of Fig. 10 shows the increase in total time (dashed dotted) when using different time constants. For example, using the time constant \( \tau = 50 \) s, the total time increases 24% from the original cycle duration (Table 2) for the aggressive cycle and 7% for the medium cycle.

In fact, the right column of Fig. 10 shows the average FCS efficiency during the original cycle duration (dashed) and the total average FCS efficiency, including the extra time used to recharge the battery (solid). For the mild cycle, the FCS operation after the cycle duration decreases the average FCS efficiency. Though, for the aggressive cycle the FCS operation after the cycle duration increases the average FCS efficiency. This explains why the total energy loss increases when increasing the time constant for the mild cycle and that an increase in time constant for the aggressive cycles lowers the overall energy loss.

Furthermore, the total hydrogen energy loss (separated into energy loss due to added mass (\( \Delta m \)), energy expended in the battery (\( \text{btt} \)), energy loss in the FCS during the cycle’s duration (\( \text{fcs} \)) and the total energy loss in the FCS (\( \text{fcs} \))) for different time constants, using, respectively, the cycle’s optimal battery sizes \( n_{\text{opt}}(\tau) \) (Fig. 9), is shown in the left column of Fig. 10. Note that the left column of Fig. 10 corresponds to the lines \( n_{\text{opt}}(\tau) \) in Fig. 9. The energy loss due to the added mass (\( \Delta m \)) is now constant because of the fixed battery sizes. Figure 10 shows that the expended energy in the battery decreases slightly when increasing the time constant for all three cycles. The FCS energy loss at first decreases when increasing the time constant for all three cycles. However, the FCS energy loss increases for large time constants for the mild cycle and decreases for large time constants due to the different FCS efficiencies during the cycle and the period of recharging after the cycle.

4.2 Comparison to Dynamic Programming Optimizations.

The effects of the battery size to the system efficiency when using the PC instead of DP differs in two fundamental ways. First, as shown in Fig. 11 (left column), the FCS efficiency does not increase when increasing the battery size. The reference current distribution (right column) is relatively the same for the different battery sizes. The constant FCS efficiency and reference current distribution is caused by the fact that the PC is designed to use the FCS as much as possible, without violating the OER constraints, and will therefore not, as the DP, exploit the opportunity of using the battery to increase the FCS efficiency.

Second, the minimal battery size with which the controller (i) was able to control the SoC and (ii) maintain an OER higher than the limit \( \Delta \text{O}_2 \lim \) without exceeding the maximum output power of the
battery, is larger than the one specified by the DP (see Sec. 3.2). Both for the mild NYCC cycle and the medium FTP-72 cycle the minimum battery size when using the proportional controller is nine modules. For the aggressive SFTP cycle, the minimum battery size is 24 modules. The DP, however, managed to control the SoC for a battery pack with three modules, due to OER values below \( \Delta V^\text{lim} \) for a smaller battery sizes (Fig. 5) for both the mild and the medium cycle. For the aggressive cycle, using DP, the minimum size is 13 modules for the same reason.

Figure 12 shows the average energy losses, like in Sec. 3.2, separated in losses due to added mass, energy expended in the battery, and remaining losses in the FCS when the proportional controller controls the SoC. Note that Fig. 12 corresponds to \( \tau_c = 50 \) s in Fig. 9. The expended energy in the battery (bt), which, as in Sec. 3.2, decreases with increasing battery size because large battery sizes are more efficient than small when handling the same amount of power. The energy loss due to the added mass (\( \Delta m \)) is similar as in Sec. 3.2 because the corresponding electric energy loss does not depend on the controller type.

Even though the minimum sizes are larger than those using DP, there are optimal sizes for all three cycles. For the mild cycle, the optimal battery size is 18 modules, 13 modules for the medium cycle, and 26 modules for the aggressive cycle. These optimal sizes are caused, as in Sec. 3.2, by the large energy expended in the battery for relative small battery sizes and by the increasing energy loss due to the added weight for larger sizes.

The larger optimal sizes, when using the proportional controller, are due to the fact that the simple controller does not have any knowledge of the future at all. Hence, the controller can not maintain the reference current on a high level to prepare for future large increases in power demand. Figure 13 shows a time plot of the acceleration power demand, reference current, and state of charge when DP (solid), and the PC (dotted) are controlling the state of charge during a portion the medium cycle. The reference current profiles shows that DP reacts earlier (gray areas), even though the OER is limited, than the PC for increasing power demands. In fact, the DP increases the FCS reference current before the acceleration power demand increases due to the predictive nature of the DP strategy. This explains why DP is able to utilize smaller batteries than the PC, thus the different minimum battery sizes.

One improvement of the PC could be to not only limit the positive FCS reference currents changes but also including a similar OER map, Fig. 2, for negative transients. This could prepare the PC for increases in the FCS reference current and produce a more load-leveling profile.

Because the proposed controller is very simple and the DP is an optimal controller, a suggestion of a range of battery sizes can be made for each cycle. For the mild cycle, the optimal battery size range from 15 to 18 modules, for the medium cycle 7–13 modules, and for the aggressive cycle 15–26 modules.

Note that for the mild cycle, the optimal sizes only differ three modules while the difference for the medium cycle is six modules and 11 modules in the aggressive cycle. This is an indication of that, for the mild cycle, the optimal battery size is somewhat independent of the controller while there is, for the medium and definitely for the aggressive cycles an opportunity for battery downsizing when optimizing the controller.

5 Conclusions and Future Work

This paper shows that, for the considered vehicle, hybridization could be beneficial in terms of reduced hydrogen consumption when excursions in OER from a nominal value is penalized. First, the dynamic programming approach suggests an optimal battery size for every drive cycle. Second, the battery sizing when using the proposed nonlinear causal controller shows that the optimal battery sizes is larger than when using DP. However, the optimal battery size differs less for the mild cycle than for the medium and the aggressive cycles, indicating that the hybridization needs for the mild cycle could be independent of the control strategy.

The optimal battery size depends on the cycle, the vehicle weight, the type of battery, and energy density of the battery. Thus, this study is very specific to our configuration, the specific weights, and characteristics of the vehicle and its different components. Furthermore, the cost function when using DP influences, in general, the optimal results and, in particular, the optimal battery sizes in this study. The model used in this study is very simple and does not consider mechanical losses in the power train. The results can however be seen as an indication of how the optimal battery size change when considering the FCS transient loading compared to when disregarding the OER [1].

A stochastic dynamic programming approach will provide a causal stochastic optimal control law, for a given stochastic drive-cycle model, which could provide an interesting comparison to the
deterministic DP and the fixed structure controller results. In future work, this study could be performed under fixed vehicle net power by resizing battery and FCS, simultaneously. Resizing the FCS depend on scalable FCS auxiliary losses and associated transient response.

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Appendix: Dynamic Programming Algorithm
To optimally control

\[ x_{k+1} = f(x_k, u_k, w_k) + x_k \]  

(A1)

with the state \( x_k \) = SoC, the reference current as the input \( u_k = I_{ref} \), and the power demand as the disturbance \( w_k = P_{dem} \). The variables \( x_k, u_k, \) and \( w_k \) are limited to the finite spaces \( X, U, \) and \( W \) with \( x_k \in X, u_k \in U, w_k \in W \). The DP algorithm allows us to find the control sequence \( u = (u_0, \ldots, u_{K-1}) \) that minimizes the cost function

\[
\sum_{t=0}^{T} J(x_t, u_t) \rightarrow \min
\]

(A2)

where \( J(x_t, u_t) \) is the cost to use the input \( u_t \) at the state \( x_t \). The cost \( J \) is defined as \( J = (a\Delta\text{SoC})^2 + \beta W_{H_2} \), where \( \Delta\text{SoC} \) is the deviation of the battery’s state of charge from the reference value \( \text{SoC}_{ref} \), \( W_{H_2} \) is the hydrogen consumption in the FCS, and \( \{a, \beta\} \) are weights.

To solve the optimal control problem (A2), we need to define an intermediate problem that starts at time \( k \) with an initial state \( x_k \). Let us define \( V(x_k, k) \) the optimal cost to go or value function that will be incurred if the system starts at state \( x_k \) at time \( k \) and continues to the final time \( K \).

\[
V(x_k, k) = \min_{u(k), (k+1) \ldots (K-1)} \sum_{t=k}^{T} J(x_t, u_t, w_t)
\]

(A3)

with the final penalty \( V(x_K, K) = (\Delta\text{SoC})^2, \Delta = 10^3 \). The final state \( x_K \) at time \( K \) is penalized to ensure that the final state of charge is close to the initial state of charge. We then obtain a relationship that relates the value function at a certain point in time to the value function at a later point in time. Let \( x_m \) be the state at time \( m \), and suppose that \( (u_{m_0}, u_{m+1}, \ldots, u_{K-1}) \) is a given control sequence that generates the trajectory \( x_{m_0}, x_{m+1}, \ldots, x_K \). Let \( l \) be the anytime instance that satisfies \( m<l \leq K-1 \). The value function satisfies

\[
V(x_m, m) = \sum_{l=m}^{K-1} J(x_l, u_l) + V(x_l, l) \rightarrow \min_{u(m), \ldots, u(l-1)} \times \sum_{l=m}^{K-1} J(x_l, u_l) + V(x_l, l)
\]

(A4)

which is the principle of optimality. In other words if \( u_{m}, \ldots, u_{l}, u_{l+1} \) is optimal for the problem starting at \( k=m \) and \( x_m, x_{m+1}, \ldots, x_{l} \) is the resulting trajectory, then \( u_{m}, \ldots, u_{K-1} \) is optimal for the problem that starts at \( k=l \) with initial condition \( x_{l} \). Finally, consider \( m=k-1 \) and \( l=k \) then the Bellman equation [2] is obtained

\[
V(x_{k-1}, k-1) = \min_{u_{k-1}} [J(x_{k-1}, u_{K-1}) + V(x_k, k)]
\]

(A5)

which allows the calculation of the optimal control sequence backward starting from \( k=K \). For more technicalities on the algorithm and the grid see [11].

References